SHOCK-WAVE PASSAGE THROUGH AN INHOMOGENEOUS MEDIUM

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The passage of a medium-power shock wave through a condensed medium with smoothly changing density is considered (initial speed of sound is varied). The solutions obtained describe the thermodynamic parameters of the medium behind the shock-wave front.

Propagation of shock waves in inhomogeneous media has been studied by a number of authors [1-7]. In those studies the equation of state of the medium was chosen in the simplified form of that for an ideal gas. However, it is of physical and practical interest to study the rules of shock-wave propagation in condensed media with variable characteristics.

1. Equation of State

The present study will examine media with a Landau — Stanyukovich-type equation of state [8], which allows variation of both density and the speed of sound in the condensed material through which the wave propagates. At moderate pressures in the shock wave (~10^6 atm) the dependence of energy and pressure on temperature and density may be written in the form of two components, one of which is related solely to elastic forces, while the other is connected solely with thermal motion [9]. They are usually termed the "cold" (E_c and P_c) and "thermal" (E_t and P_t) components. The thermal pressure P_t is proportional to temperature T (with T far exceeding the initial temperature of the material before compression) and inversely proportional to specific volume V. The proportionality coefficient Γ (Gruneisen coefficient), generally speaking, is density dependent. However, this dependence may be neglected [9]. We take $\Gamma = 2$, which corresponds to a number of metals at slight compression. For the cold pressure we use the function proposed in [10]:

$$P_c = a\left[\left(\frac{V_0}{V}\right)^3 - 1\right], \tag{1.1}$$

where, as Gandel'man indicated [11], $a = (1/3)\rho_0 c_0^2$. Then the total energy $E = E_c + E_t$ of the material has the form

$$E = \frac{c_0^2}{6V_0V^2}(V_0 - V)[V_0^2 + VV_0 - 2V^2] + 3RT. \tag{1.2}$$

2. Shock Adiabat and Isentrope

The equation of state chosen gives very simple expressions for the shock adiabat. On the wave front we obtain the function for the change in pressure amplitude

$$P_f = \frac{c_0^2}{V_0} \cdot \frac{V_0 - V_f}{2V_f - V_0} \tag{2.1}$$

and the temperature change

$$T_f = \frac{c_0}{18R} \cdot \frac{(V_0 + V_f)(V_0 - V_f)^3}{V_0V_f(2V_f - V_0)}. \tag{2.2}$$

Behind the wave front the motion of the material is isentropic. Since $TV^2 = T_fV_f^2$, the equation of the isentrope relating values on the front to values within the volume encompassed by the shock wave takes on the

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form

$$c^2 = c_0^2 \frac{V_f^3}{V^2 V_0} \cdot \frac{2V_0 - V_f}{2V_f - V_0}. \tag{2.3}$$

3. Basic Equations

Since a shock wave propagating through a material with variable density creates a variable entropy at each mass element, it will be convenient to solve the problem in Lagrange variables.

With the above assumptions the system of equations for the problem* will have the form

$$\frac{\partial U}{\partial t} = -\frac{\partial P}{\partial m}, \tag{3.1}$$

$$\frac{\partial V}{\partial t} = \frac{\partial U}{\partial m}, \tag{3.2}$$

$$P = \frac{c_0^2}{3V_0}\left[\left(\frac{V_0}{V}\right)^3 \frac{2V_0 - V_f}{2V_f - V_0} - 1\right]. \tag{3.3}$$

As boundary conditions on the shock wave we take the equations for velocity of the material

$$U_f = c_0\left(1 - \frac{V_f}{V_0}\right)\left(2\frac{V_f}{V_0} - 1\right)^{-\frac{1}{2}} \tag{3.4}$$

and for the front trajectory in Lagrangian form

$$\frac{dm_f}{dt} = \frac{c_0}{V_0}\left(2\frac{V_f}{V_0} - 1\right)^{-\frac{1}{2}}. \tag{3.5}$$

We assume that the shock wave in the medium is excited by detonation of a very thick layer of explosive substance attached to one side. The detonation wave striking the boundary of the medium m = 0 is reflected and propagates in the opposite direction from the explosion products. Because the explosion products form a thick layer, on the boundary with the compressed material a constant pressure is maintained. Thus, at m = 0, P = const, and, consequently, the specific volume is constant. From continuity conditions for the value of this pressure we have

$$0.578 - \frac{P}{P_0}\sqrt{\frac{V_0}{V_0^1}\left(\frac{c_0^2}{P_0 V_0} + 2\frac{P}{P_0}\right)} = \sqrt{\left(\frac{P}{P_0} - 1\right)\left[1 - \left(\frac{P_0}{P}\right)^{\frac{1}{3}}\right]}. \tag{3.6}$$

which was obtained by use of the simplest form of the equation of state for the detonation products [12].

4. Asymptotic Solution for Initial Stage of Process

To estimate the character of the change in the shock wave upon propagation through the medium of inhomogeneous density we use an expansion of the dependent variables for small m and t in the form of a power series in m and t. We assume that

$$V_0 = V_{00}(1 + \alpha m + \beta m^2), \tag{4.1}$$

$$c_0 = c_{00}(1 + \varkappa m + \delta m^2), \tag{4.2}$$

where V_{00}, α, β, c_{00}, $\varkappa$, δ are some arbitrary constants. We seek the expansions near m = 0 in the form

$$V = \tilde{V}(1 + V_1 m + V_2 m^2 + V_4 mt), \tag{4.3}$$

$$U = \tilde{U}(1 + U_1 t + U_2 m^2 + U_3 t^2 + U_4 mt), \tag{4.4}$$

$$t_f = \frac{m_f}{D}(1 + D_1 m_f). \tag{4.5}$$

To determine the coefficients of the expansion we substitute Eqs. (4.3) and (4.4) in Eqs. (3.1)-(3.3) and use boundary conditions (3.4)-(3.6).

*The brief paper by Kompaneets, Romanova, and Yampol'skii in Zh. Éksp. Teor. Fiz., Pis'ma Red., [16, No. 4, 259 (1972)] unfortunately presents these equations in an erroneous form, as was indicated to the authors by V. N. Svidinskii.

The expressions for pressure, mass velocity, and shock-front velocity at m = 0 are written in terms of the constant initial compression value $\theta = (V_{00}/\tilde{V})$:

$$\tilde{P} = c_{00}^2 \rho_{00} \frac{\theta - 1}{2 - \theta}, \tag{4.6}$$

$$\tilde{U} = c_{00} \frac{\theta - 1}{\sqrt{\theta(2 - \theta)}}, \tag{4.7}$$

$$\tilde{D} = c_{00} \rho_{00} \sqrt{\frac{\theta}{2 - \theta}}. \tag{4.8}$$

The coefficients of the linear terms, written in terms of the dimensionless parameter $\pi = (\theta - 1)/(1 - 2\theta)$, have the form

$$U_{1D} = D_1 = \alpha \frac{\pi + 1}{3\pi + 2} - \varkappa \frac{1}{3\pi + 2}, \tag{4.9}$$

$$V_1 = \alpha \frac{3\pi^2 + 4\pi + 2}{(\pi + 1)(3\pi + 2)} + \varkappa \frac{3\pi}{(\pi + 1)(3\pi + 2)}. \tag{4.10}$$

The expressions for coefficients of the quadratic terms have a more cumbersome form and therefore will not be presented.

Results of calculations performed with the relationships obtained here permit the following general conclusions. Upon shock-wave propagation in a medium with decreasing density the pressure at the wave front drops, and the change in velocity of sound may reach the energy output on the constant asymptote. Also, in the case of decreasing density, compression in the wave increases slightly, and with increase in the initial speed of sound the opposite occurs; analogous results are obtained for the temperature T_f and the ratio P_t/P_c at the front.

The mass velocity U_f for the case of falling density increases at any c_0.

When the shock-wave propagates through a material with increasing initial density ρ_0 the pressure at the front P_f increases if c_0 is constant or increases. For falling velocity of sound, although the dependence on c_0 is weaker than that on ρ_0, the pressure P_f may fall. Moreover, for increasing ρ_0 the mass velocity U_f, compression in the wave, temperature T_f, and P_t/P_c on the front all fall with wave propagation. A drop in initial velocity of sound affects the results negatively.

Most interesting from the viewpoint of change in thermodynamic and mechanical parameters in a material disturbed by a shock wave are media with constant acoustic impedance ($\rho_0 c_0 = \text{const}$).

Comparison of the results with data of numerical solution by computer indicated that for some values the solutions obtained by use of the expansion are quite accurate and, on the whole, do give a proper indication of the tendencies of parameter variation in condensed materials disturbed by a shock wave.

NOTATION

R, gas constant, referred to 1 g of substance; Γ, Gruneisen coefficient; m, mass coordinate; t, time coordinate; t_f, time at which shock wave reaches mass m_f; U, mass velocity; U_f, mass velocity at front; D, shock-front velocity; V, specific volume; V_0, initial specific volume; V_f, specific volume of compressed material at front; P, pressure; P_t, thermal pressure; P_c, cold pressure; P_f, pressure at front; E_t, thermal energy; E_c, cold energy; T, temperature; T_f, temperature of material at front; c, velocity of sound; c_0, velocity of sound in material before compression; c_f, velocity of sound at front; ρ_0, density of uncompressed material.

LITERATURE CITED

1. G. M. Gandel'man and D. A. Frank-Kamenetskii, Dokl. Akad. Nauk SSSR, 107, 811 (1956).
2. A. S. Kompaneets, Dokl. Akad. Nauk SSSR, 130, No. 5 (1960).
3. É. I. Andriankin, A. M. Kogan, A. S. Kompaneets, and V. P. Krainov, Zh. Prikl. Mekh. Tekh. Fiz., No. 6 (1962).
4. Yu. P. Raizer, Dokl. Akad. Nauk SSSR, 153, 551 (1963).
5. Yu. P. Raizer, Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1964).
6. W. Chester and R. Collins, Isr. J. Technol., 8, 345 (1970).

7. R. Collins and Chen Hsiang-teh, Lect. Notes Phys., 8, 264 (1971).
8. L. D. Landau and K. P. Stanyukovich, Dokl. Akad. Nauk SSSR, 46, 399 (1945).
9. Ya. B. Zel'dovich and Yu. P. Raizer, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena, Academic Press (1966-1967).
10. K. P. Stanyukovich, Transient Motions of a Solid Medium [in Russian], Gostekhizdat, Moscow (1971).
11. G. M. Gandel'man, Author's Abstract of Doctoral Dissertation, Moscow (1966).
12. Ya. B. Zel'dovich and A. S. Kompaneets, Detonation Theory [in Russian], Gostekhizdat, Moscow (1955).

EFFECT OF LATTICE OXYGEN ON THERMAL AND ELECTRICAL PROPERTIES OF FLUORITE

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The electrical conductivity, thermal conductivity, and thermo-emf of CaF_2 crystals with an oxygen impurity are measured. Values of the defect transfer parameter are obtained. The existence of thermal diffusion of oxygen-vacancy complexes in fluorite is proposed.

One of the greatest difficulties in synthesizing fluorite crystals is elimination of oxygen, which is always present in the original ingredients. This problem arises because of the direct connection between optical properties of CaF_2 and O^{2-} concentration. In particular, the presence of oxygen produces unsatisfactory radiation stability in the crystals, low transparency in the ultraviolet, existence of light-scattering phases, etc. At the same time, in growth of CaF_2 crystals activated by rare earth metals for use as volume registration devices oxygen plays an important role, since it ensures stability of the nonequilibrium transition of the rare earth ion from the trivalent to the bivalent state [1]. Thus, it is of practical importance to study means for determination and control of oxygen content in fluorite specimens.

In contrast to univalent cation impurities which are able to produce both vacancies and interstitial ions (depending on the mode of solution in the fluorite lattice), the presence of O^{2-} leads only to vacancies [1]. Thus, the coincidence of kinetic defect parameters in oxygen-containing crystals and specimens of CaF_2 + MeF is additional and independent confirmation of the vacancy nature of the defects produced by a univalent metal impurity (as in the case of BaF_2 [2]). Interest in the anion impurity in anti-Frankel systems is caused by the fact that in this case conditions exist for thermal diffusion of the impurity ion or its complexes.

The present study considers the electrical conductivity, thermal conductivity, and thermo-emf of a number of oxygen-containing CaF_2 specimens. The measurement method is described in [4, 5]. The specimens were produced by the Stokebarger method in a graphite crucible. Doping with oxygen was effected by admission of a weak air flow through the vacuum crystallization chamber over the course of the entire crystal-growth process. As a result, the transmission spectra of the crystals (Fig. 1) show intense absorption bands at ~155 nm and ~205 nm, connected with the presence of O_2 [3]. For comparison, Fig. 1 also shows the transmission spectrum of pure CaF_2. In both specimens (Nos. 1 and 6, Fig. 2a) the oxygen content was determined to an accuracy of 20% by the vacuum fusion method (No. 1, 0.22 mole % O^{2-}; No. 6, 0.64 mole % O^{2-}).

Results of conductivity measurements on several $CaF_2 + O_2$ specimens (Nos. 1-6, Fig. 2a) reveal that the temperature dependences $\sigma = f(1/T)$ are similar to each other and to analogous functions for crystals of CaF_2 + NaF (No. 7, Fig. 2a). On the curves $\sigma = f(1/T)$ one can distinguish segments of natural conductivity (high-temperature region), dissociation (middle region), and association (low-temperature region). The slopes of these segments characterize the activation energies, which, within the limits of experimental error, coincide with the following values for CaF_2 + NaF [4]: in the dissociation region $h_- = (0.5 \pm 0.03)$ eV, in the association region $(h_- + (1/2)h_{a^-}) = (0.7 \pm 0.05)$ eV. Hence, the bonding energy of the O^{2-}-vacancy h_{a^-} complexis

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